Fault-Tolerant k-Supplier with Outliers

Deeparnab Chakrabarty, Luc Cote, **Ankita Sarkar**

(Remote) Talk at Purdue Theory Seminar
November 10, 2023

The Problem

Fault-tolerant k-Supplier with Outliers (FkSO)

Background: k-Supplier

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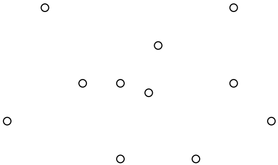

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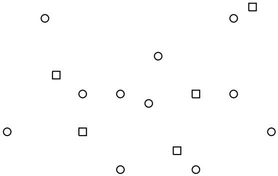

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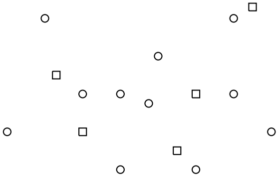
- Metric d on $C \cup F$



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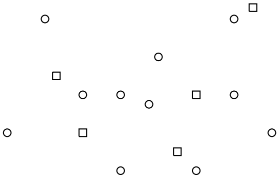

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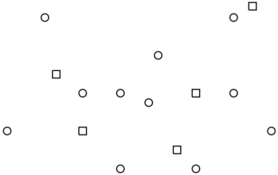

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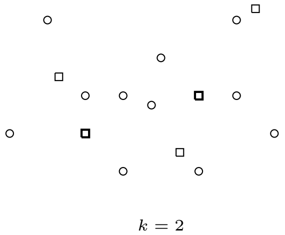

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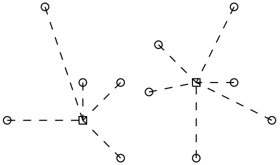

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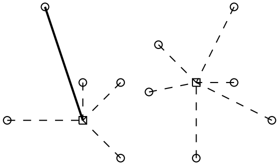

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Tight 3-approximation [Hochbaum and Shmoys, 1986].

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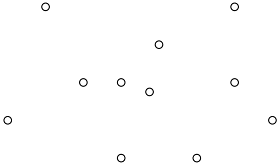

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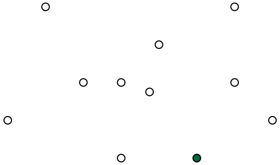

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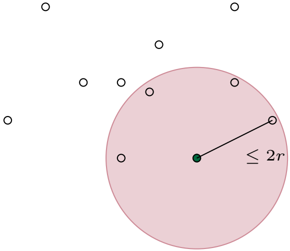

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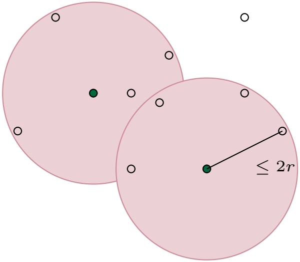

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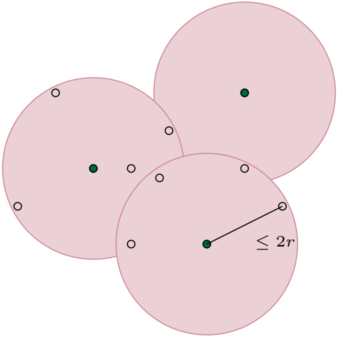

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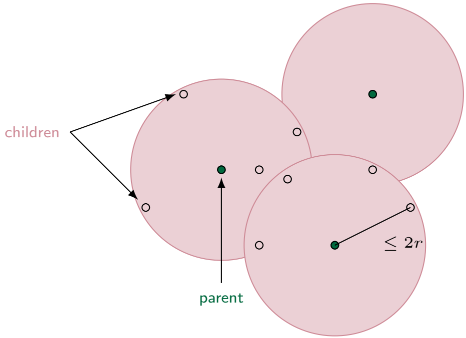

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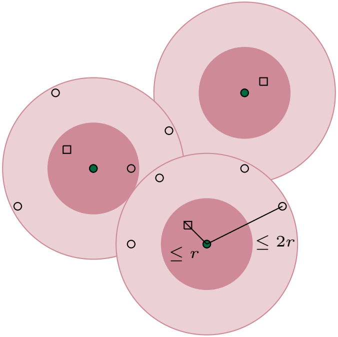

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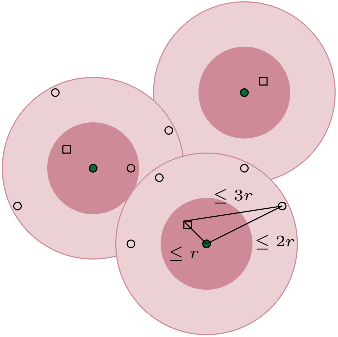

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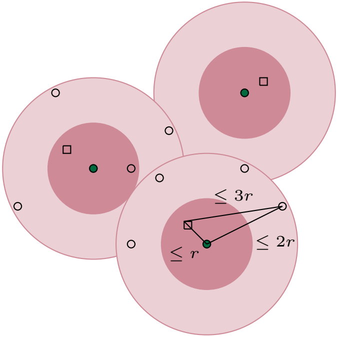

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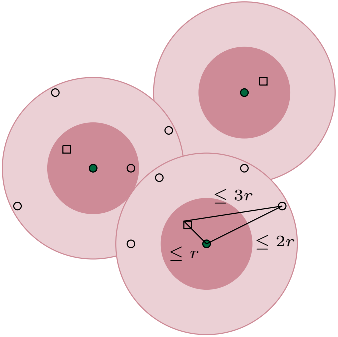

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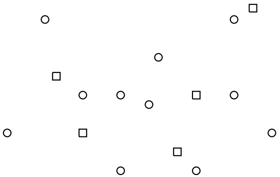

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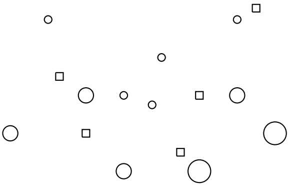

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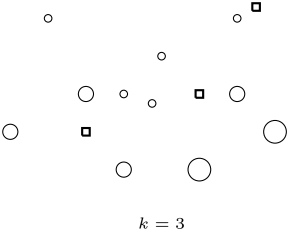

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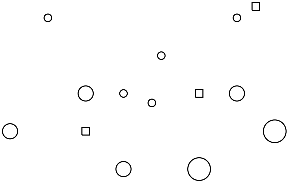

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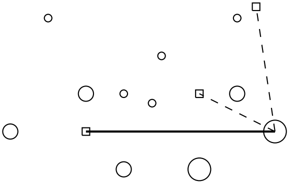

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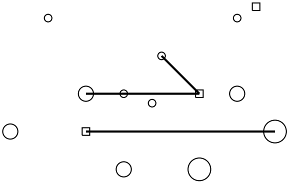

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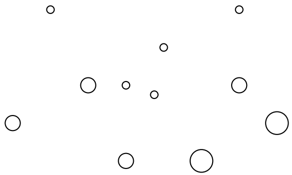
Algorithm. Carve balls in decreasing order of ℓ_v's.



- ℓ_j facilities in each $B(j, r)$
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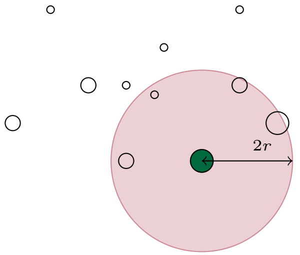

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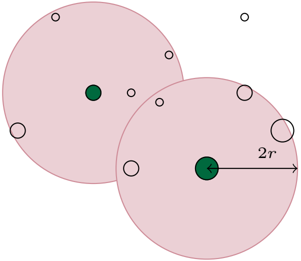

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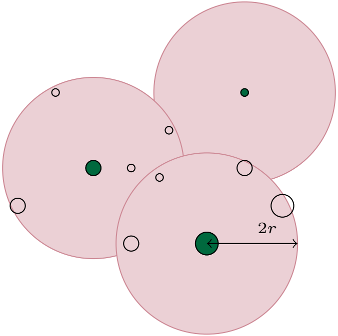

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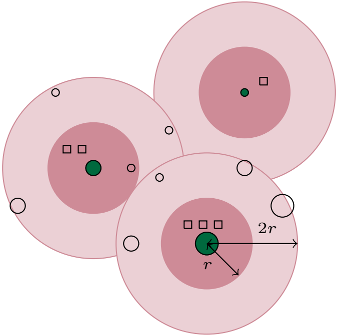

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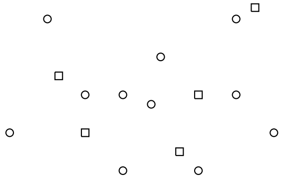

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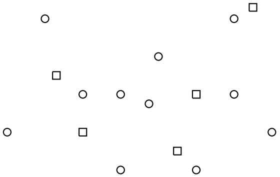

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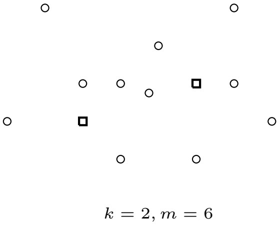

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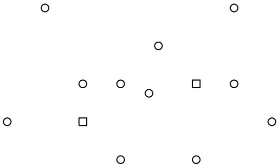

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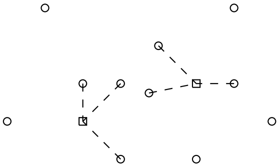

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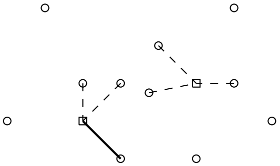

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3-approximation [Chakrabarty, Goyal, and Krishnaswamy, 2016]

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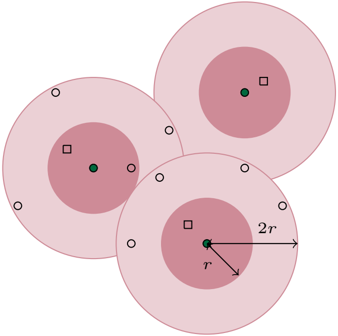

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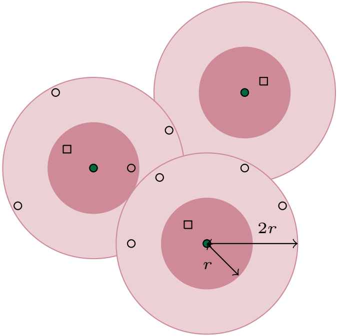

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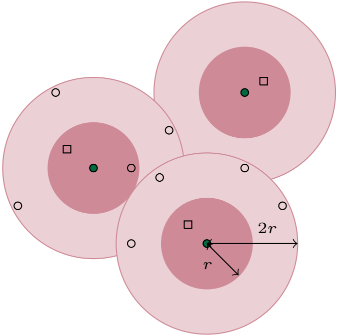

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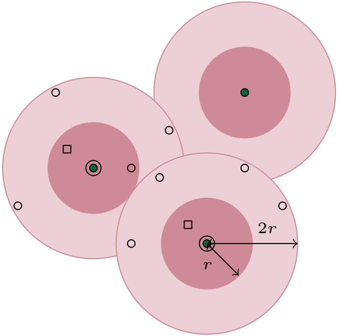

- $\forall j \in R, \forall v \in \text{child}(j), \text{cov}_j \geq \text{cov}_v$
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- LP and above $\implies |T| \geq m$

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- Enough inliers?
- Want: k largest $|\text{child}(j)|$'s sum to $\geq m$
- $\sum_{j \in R} |\text{child}(j)| \, \text{cov}_j \geq \sum_{j \in R} \sum_{v \in \text{child}(j)} \text{cov}_v$
  $= \sum_{v \in C} \text{cov}_v \geq m$
- LP and above $\implies |T| \geq m$
- **Takeaway: decreasing order of cov_v's.**

The Problem

Fault-tolerant k-Supplier with Outliers

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- $S \subseteq F$, $|S| \leq k$
- $\underbrace{T}_{\text{inliers}} \subseteq C$, $|T| \geq m$, minimize $\max_{v \in T} d_{\ell_v}(v, S)$

FkSO: Natural LP

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- $\sum_{v \in C} \mathsf{cov}_v \geq m$, $\sum_{i \in F} x_i \leq k$
- $\forall v \in C$, $\sum_{i \in B(v,r) \cap F} x_i \geq \ell_v \mathsf{cov}_v$

FkSO: Infinite integrality gap!

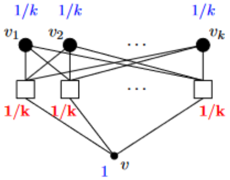

Figure 1: *One of the k identical gadgets in the gap example, showing LP values in* **red** *(x values) and blue (*cov *values). The "edges" represent distance* 1*, and all other distances are determined by making triangle inequalities tight. The fault-tolerances are* $\ell_{v_1} = \ell_{v_2} = \cdots = \ell_{v_k} = k$*, and* $\ell_v = 1$.

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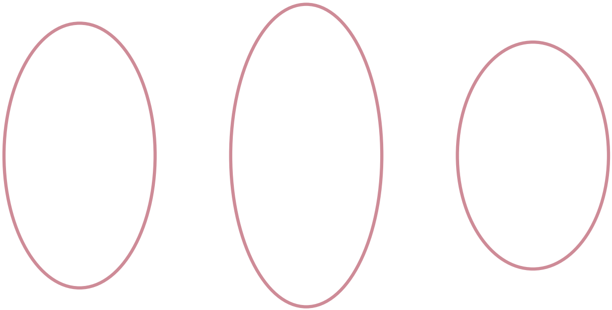
- Gap example for weaker LP had opposite orders

Good partition



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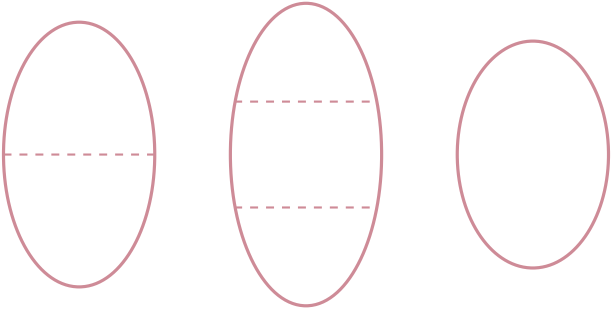
Good partition



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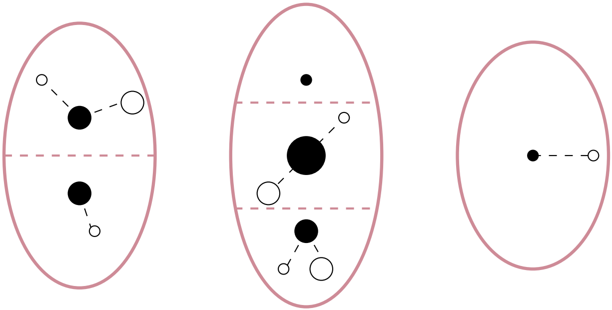

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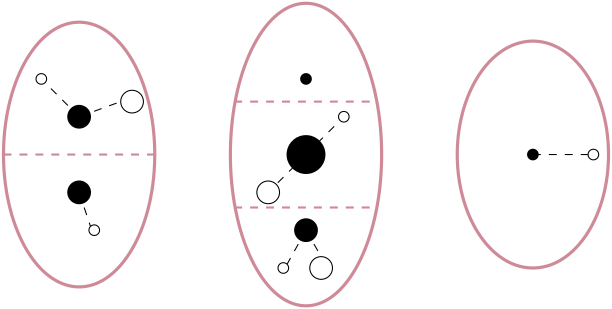

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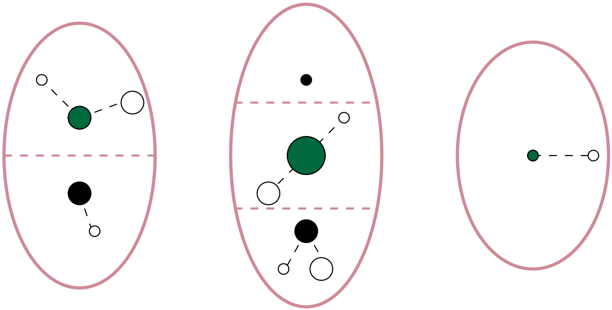

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3. Every $P \in \mathcal{P}$ has radius ρr around $j_P := \text{argmax}_{v \in P}\, \ell_v$

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Theorem

Given cov_v*'s from the LP, if there exists a* (ρ, cov)*-good partition, then there is a* $(\rho + 1)$*-approximation.*

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Hence $(4t - 1)$-approximation.

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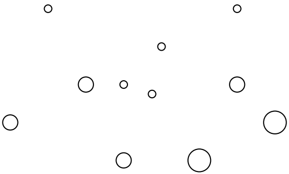

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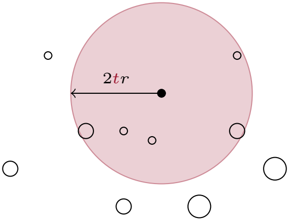

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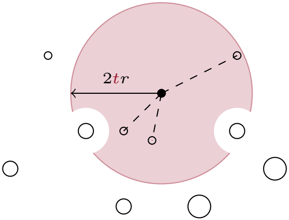

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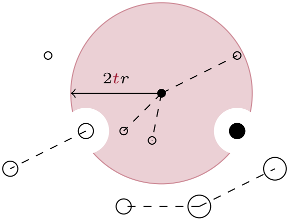

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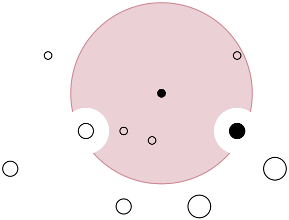

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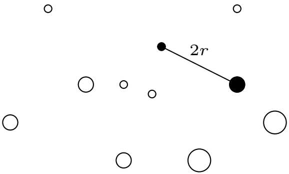

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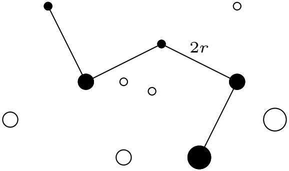

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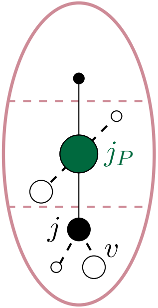

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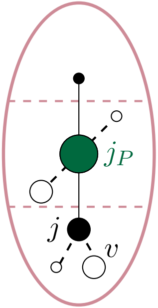

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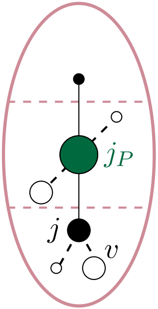

- $j_P := \text{argmax}_{v \in P} \ell_v$
- $d(v, j_P) \leq \underbrace{d(v,j)}_{2tr \because v \in \text{child}(j)} + \underbrace{d(j, j_P)}_{2(t-1)r \text{ by diam}} \leq \underbrace{(4t-2)}_{=\rho} r$

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- Else: violated LP constraint.

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- Limitation of good partition: $\rho = o(t)$ not possible.